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ALGEBRAIC FUZZY ROUGH SHEAF GROUP FORMED BY POINTED FUZZY ROUGH TOPOLOGICAL GROUP

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ABSTRACT

The purpose of this paper is to introduce the concepts of fuzzy rough homotopy and fuzzy rough sheaf group. In this connection, some interesting properties of fuzzy rough sheaf are discussed.

KEYWORDS: Fuzzy Rough Homotopy, Fuzzy Rough Topological Group and Fuzzy Rough Sheaf 2000 Mathematics Subject Classification 54A40 - 03E72

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [10]. Fuzzy sets have applications in many fields such as information [8] and control [9]. The theory of fuzzy topological spaces was introduced and developed by Chang [3] and since then various notions in classical topology has been extended to fuzzy topological spaces. Pawlak [7] introduced the concept of rough set. R. Biswas and S. Nanda [2] defined the notion of rough group and rough subgroup. S. Nanda [8] introduced the concept of fuzzy rough set. In this paper, the concepts of fuzzy rough homotopy and fuzzy rough topological groups are studied. In this connection, some properties of fuzzy rough sheaf are established.

2. PRELIMINARIES

Let U be any non empty set and let $\mathcal{B}$ be a complete subalgebra of the Boolean algebra $\mathcal{P}(U)$ of subsets of U. The pair $(U, \mathcal{B})$ is called rough universe. Consider a rough set $X = (X_L, X_U) \in \mathcal{B}^2$ with $X_L \subset X_U$.

Defnition 2.1.[6] A fuzzy rough set $A = (A_L, A_U)$ in X is characterized by a pair of maps $A_L : X_L \to I$ and $A_U : X_U \to I$ with $A_L(x) \leq A_U(x)$ for every $x \in X_U$. The collection of all fuzzy rough sets in X is denoted by $FRS(X)$.

Defnition 2.2. [5] Let $A = (A_L, A_U)$ be a fuzzy rough set in X. Then the *complement* A' of A is defined by ordered pairs (A'_L, A'_U) of membership functions where $A'_L(x) = 1 - A_L(x)$ and $A'_U(x) = 1 - A_U(x)$.

Definition 2.3. [5] Let $(V, \mathcal{B})$ and $(V_1, \mathcal{B}_1)$ be two rough universes $f : (V, \mathcal{B}) \to (V_1, \mathcal{B}_1)$. Let $A = (A_L, A_U)$ be a fuzzy rough set in X. Then $Y_L = f(X_L)$ and $Y_U = f(X_U)$. The *image of* A under f, denoted by $f(A) = \big(f(A_L), f(A_U)\big)$ is defined by $f(A_L)(y) = \vee\{A_L(x) : x \in X_L \cap f^{-1}(y)\}$ for every $y \in Y_L$ and $f(A_U)(y) = \vee\{A_U(x) : x \in X_U \cap f^{-1}(y)\}$ for every $y \in Y_U$. Let $B = (B_L, B_U)$ be a fuzzy rough set in Y where $Y = (Y_L, Y_U) \in \mathcal{B}_1^2$ is a rough set. Then $X = f^{-1}(Y) \in \mathcal{B}_1^2$, where $X_L = f^{-1}(Y_L)$, $X_U = f^{-1}(Y_U)$. Then the *inverse image* of B under f, denoted by $f^{-1}(B) = \big(f^{-1}(B_L), f^{-1}(B_U)\big)$ is defined by $f^{-1}(B_L)(x) = B_L(f(x))$ for every $x \in X_L$ and $f^{-1}(B_U)(x) = B_U(f(x))$ for every $x \in X_U$.

Definition 2.5. [5] The null fuzzy rough set and whole fuzzy rough set in X are defined by $\tilde{0} = (0_L, 0_U)$ and $\tilde{1} = (1_L, 1_U)$.

3. PROPERTIES OF FUZZY ROUGH HOMOTOPY

Definition 3.1. A fuzzy rough topology on a rough set X is a family T of fuzzy rough sets in X which satisfies the following conditions: (i) $\tilde{0}, \tilde{1} \in T$, (ii) If $A, B \in T$, then $A \cap B \in T$ and (iii) If $A_j \in T$ for all j$\in J$ then $\cup_{j \in J} A_j \in T$. The pair (X, T) is called a *fuzzy rough topological space*. Every member of T are called *fuzzy rough open*. The complement of a fuzzy rough open set is called *fuzzy rough closed*.

Definition 3.2. Let X be a rough set. Let $A = (A_L, A_U)$ be a fuzzy rough set in X. The set $supp\ A_L = \{x \in X_L : A_L(x) > 0\}$ and $supp\ A_U = \{x \in X_U : A_U(x) > 0\}$. Then $supp\ A = (supp\ A_L, supp\ A_U)$ is called rough support of fuzzy rough set A.

Definition 3.3. Let (X, T) be a usual rough topological space. The collection $\vec{T} = \{A : A \text{ is a fuzzy rough set on } X \text{ and } suppA \in T\}$ is a fuzzy rough topology on X, called the fuzzy rough topology introduced by T. $(X, \vec{T})$ is called fuzzy rough topological space introduced by (X, T).

Definition 3.4. Let $(X, T), (Y, S)$ be any two fuzzy rough topological spaces. A function $f : (X, T) \to (Y, S)$ is said to *be fuzzy rough continuous* if for each fuzzy rough open set W in S the inverse image $f^{-1}(W)$ is in T. Conversely, f is fuzzy rough open if for each fuzzy rough open set V in T the image $f(V)$ is in S.

Definition 3.5. Let (X, T) and (Y, S) be fuzzy rough topological spaces and (R, T^*) be usual topological space, $(I, \vec{\varepsilon})$ fuzzy rough topological space induced by (I, ε) topological space and $f, g : (X, T) \to (Y, S)$ are fuzzy rough continuous functions. If there exists a fuzzy rough continuous function $F : (X, T) \times (I, \vec{\varepsilon}) \to (Y, S)$ such that $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$ for every $x \in X_L$; $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$ for every $x \in X_U$ then F is called *fuzzy rough homotopy from f to g* and denoted by $f \simeq g$.

Note 3.1. ε always denote Euclidean rough subspace topology on $I = [0,1]$ and then $(I, \vec{\varepsilon})$ denotes the fuzzy rough topological space introduced by the usual rough topological space (I, ε).

Definition 3.6. Let (X, T) and (Y, S) be fuzzy rough topological spaces and (R, T^*) be usual topological space, $(I, \vec{\varepsilon})$ fuzzy rough topological space induced by (I, ε) topological space and $f, g : (X, T) \to (Y, S)$ are fuzzy rough continuous functions. If there exists a fuzzy rough continuous function $F : (X, T) \times (I, \vec{\varepsilon}) \to (Y, S)$ such that $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$ for every $x \in X_L$; $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$ for every $x \in X_U$ and $F(x, t) = f(x) = g(x)$ for every $x \in X_{0_L} \subseteq X_L$; $F(x, t) = f(x) = g(x)$ for every $x \in X_{0_U} \subseteq X_U$ then F is called *fuzzy rough homotopy from f to g* and denoted by $f \simeq g(rel\ X_0)$.

Definition 3.7. Let (X, T) be a fuzzy rough topological space and $A \subset X$. Then the fuzzy rough subspace topology on A is defined by $T_A = \{B/A : B \in T\}$. The fuzzy rough subspace topology on A is denoted by T_A and the pair (A, T_A) is called a fuzzy rough subspace topology on (X, T).

Definition 3.8. Let X be a rough set and $P \subset X$. Then the *fuzzy rough characteristic function* of P is defined by

$\chi_{P_L}(x) = \begin{cases} 1 \text{ if } x \in P_L \\ 0 \text{ if } x \notin P_L \end{cases}$ and $\chi_{P_U}(x) = \begin{cases} 1 \text{ if } x \in P_U \\ 0 \text{ if } x \notin P_U \end{cases}$. The fuzzy rough characteristic function of P is denoted by

$\chi_P = (\chi_{P_L}, \chi_{P_U})$.

Property 3.1. Let (X,T) and(Y,S) be any two fuzzy rough topological spaces. Let *A* and *B* be the rough subsets of *X*. Let χ_A and χ_B be two fuzzy rough closed sets in (X,T) and $A \cup B = X$. Let $f:(A,T_A) \to (Y,S)$ and $g:(B,T_B) \to (Y,S)$ be two fuzzy rough continuous functions. If $f|_{A\cap B} = g|_{A\cap B}$, then $h:(X,T) \to (Y,S)$ defined by $h(x) = \begin{cases} f(x) \text{ for } x \in A_L \\ g(x) \text{ for } x \notin B_L \end{cases}$ and $h(x) = \begin{cases} f(x) \text{ for } x \in A_U \\ g(x) \text{ for } x \notin B_U \end{cases}$ is a fuzzy rough continuous function.

Property 3.2. The relation $\simeq$ (homo topic relative X_0) is an equivalence relation.

Notation 3.1. The equivalence class of fuzzy rough homo topic relative to X_0 of *f* is denoted by $[f]$.

Definition 3.9. Let $f:(X,T) \to (Y,S)$ be a fuzzy rough continuous function. If there is a fuzzy rough continuous $f':(Y,S) \to (X,T)$ satisfies the following conditions: (i) $ff' \simeq i_Y$ (ii) $f'f \simeq i_X$ then *f* is called *fuzzy rough homotopy equivalence*. Further, if there is a fuzzy rough homo topy equivalence function between (X,T) and (Y,S), then these spaces are called *fuzzy rough homo topy equivalent spaces* or these are *same fuzzy rough homo topy type* and denoted by $(X,T) \simeq (Y,S)$.

Remark 3.1. The same fuzzy rough homo topy type equivalent spaces is an equivalence relation.

Property 3.3. If (X,T) and (Y,S) are fuzzy rough topological equivalent spaces, then (X,T) and (Y,S) are fuzzy rough homo topic equivalent spaces.

4 FUZZY ROUGH TOPOLOGICAL GROUPS

Definition 4.1. Let *X* be a non empty set and $\lambda \in (0,1]$. If $P^\lambda_{x_{0_L}}:X_L \to I$ and $P^\lambda_{x_{0_U}}:X_U \to I$ defined by $P^\lambda_{x_{0_L}}(x) = \begin{cases} \lambda \text{ for } x \in x_{0_L} \\ 0 \text{ for } x \notin x_{0_L} \end{cases}$ for every $x \in X_L$ and $P^\lambda_{x_{0_U}}(x) = \begin{cases} \lambda \text{ for } x \in x_{0_U} \\ 0 \text{ for } x \notin x_{0_U} \end{cases}$ for every $x \in X_U$ then $P^\lambda_{x_0} = (P^\lambda_{x_{0_L}}, P^\lambda_{x_{0_U}})$ is called a *fuzzy rough point* on *X*. Here $x_0 = (x_{0_L}, x_{0_U})$ is called rough support of $P^\lambda_{x_0}$ and λ is called value of $P^\lambda_{x_0}$.

Definition 4.2. Let (X,T) be a fuzzy rough topological space. Let $A = (A_L, A_U)$ be a fuzzy rough set. Then A is said to be a *fuzzy rough neighbourhood* of a fuzzy rough point $P^\lambda_{x_0}$ iff there exists fuzzy rough open set $V = (V_L, V_U)$ such that $P^\lambda_{x_{0_L}} \in V_L \leq A_L$ and $P^\lambda_{x_{0_U}} \in V_U \leq A_U$.

Definition 4.3. Let $A = (A_L, A_U)$ be any fuzzy rough set and $P^\lambda_{x_0} = \left(P^\lambda_{x_{0_L}}, P^\lambda_{x_{0_U}}\right)$ be any fuzzy rough point. A fuzzy rough point $P^\lambda_{x_0}$ is said to be *fuzzy rough quasi coincident* with *A*, denoted by $P^\lambda_{x_0} qA$ (that is, $P^\lambda_{x_{0_L}} qA_L$ and $P^\lambda_{x_{0_U}} qA_U$) iff $x \in X_L$, $\lambda + A_L(x) > 1$ and $x \in X_U$, $\lambda + A_U(x) > 1$.

Definition 4.4. Let (X,T) be a fuzzy rough topological space. Let A be a fuzzy rough set. Then *A* is said to be an *fuzzy rough Q neighbourhood* of $P^\lambda_{x_0}$ iff there exists fuzzy rough open sets B such that $P^\lambda_{x_0} qB$ and $B \subseteq A$.

Definition 4.5. Let $(X,*)$ be a rough group. If $A, B \in FRS(X)$ and *C*, *D* are rough subsets of *X*, then $A \cdot B \in FRS(X)$, $A^{-1} = (A_L^{-1}, A_U^{-1}) \in FRS(X)$, $C * D$ and $C^{-1} = (C_L^{-1}, C_U^{-1})$ are rough subsets of *X* are defined as following type: $(A_L \cdot B_L)(x) = \sup\{min\{A_L(x_1), B_L(x_2), x_1 * x_2 = x\}\}$ for every $x, x_1, x_2 \in X_L$ and $(A_U \cdot B_U)(x) = \sup\{min\{A_U(x_1), B_U(x_2), x_1 * x_2 = x\}\}$ for every $x, x_1, x_2 \in X_U$; $A_L^{-1}(x) = A_L(x^{-1})$ for every $x \in X_L$ and $A_U^{-1}(x) = A_U(x^{-1})$ for every $x \in X_U$; $C_L * D_L = \{c * d : c \in C_L, d \in D_L\}$ and $C_U * D_U = \{c * d : c \in C_U, d \in D_U\}$ and

$C_L^{-1} = \{c^{-1}: c \in C_L\}$ and $C_U^{-1} = \{c^{-1}: c \in C_U\}$.

Definition 4.6. Let $(X,*)$ be a rough group. Let (X,T) be a fuzzy rough topological space. A traid $(X,*,T)$ is called a *fuzzy rough topological group* if it satisfies the following conditions: (i) For all $x_L, y_L \in X_L$ and $x_U, y_U \in X_U$ and any fuzzy rough open *Q* neighbourhood W of the fuzzy rough point $P_{(x*y)}^{\lambda} = \left(P_{(x*y)_L}^{\lambda}, P_{(x*y)_U}^{\lambda}\right)$ there are fuzzy rough open *Q* neighbourhoods *U* and *V* of $P_x^{\lambda} = \left(P_{x_L}^{\lambda}, P_{x_U}^{\lambda}\right)$ and $P_y^{\lambda} = \left(P_{y_L}^{\lambda}, P_{y_U}^{\lambda}\right)$, respectively such that $R \cdot V \subseteq W$. (ii) For all $x_L \in X_L$ and $x_U \in X_U$ and any fuzzy rough open *Q* neighbourhoods $V' = (V_L', V_U')$ of $P_{x^{-1}}^{\lambda} = \left(P_{x_L^{-1}}^{\lambda}, P_{x_U^{-1}}^{\lambda}\right)$ there exists a fuzzy rough open *Q* neighbourhoods *U* of $P_x^{\lambda} = \left(P_{x_L}^{\lambda}, P_{x_U}^{\lambda}\right)$ such that

$R^{-1} \subseteq V'$.

Notation 4.1. $FRC(Y,Z)$ be the set of all fuzzy rough continuous functions from (Y,S) to (Z,R).

Property 4.1. Let (Y,S) be a fuzzy rough topological space, $(Z,*'',R)$ be fuzzy rough topological group and $f,g \in FRC(Y,Z)$. If $f,g \in FRC(Y,Z)$, then $f \odot g: (Y,S) \to (Z,R)$ and $f^{-1}: (Y,S) \to (Z,R)$ $(f \odot g)(y) = f(y) *'' g(y)$ for every $y \in Y_L$ and $(f \odot g)(y) = f(y) *'' g(y)$ for every $y \in Y_U$, $f^{-1}(y) = (f(y))^{-1}$ for every $y \in Y_L$ and $f^{-1}(y) = (f(y))^{-1}$ for every $y \in Y_U$ are fuzzy rough continuous functions.

Definition 4.7. Let *X* be a rough set, $r \in [0,1]$ and $r^* = (r_L^*, r_U^*) \in FRS(X)$. For all $x \in X_L$ and $x \in X_U$, $r_L^*(x) = r$ and $r_U^*(x) = r$ defined by $r^* = (r,r)$ which is fuzzy rough set and *T* subset of *FRS*(*X*). If *T* satisfies following conditions, then *T* called a *fully stratified fuzzy rough topological space* : (i) $r^* \in T$ (ii) If $A, B \in T$, then $A \cap B \in T$ and iii) If $A_j \in T$ for all j$\in J$ then $\cup_{j \in J} A_j \in T$.

Property 4.2. Let (Y,S) be a fully stratified fuzzy rough topological space and $(Z,*'',R)$ fuzzy rough topological group. Then the pair $(FRC(Y,Z), \odot)$ is group.

Property 4.3. Let (Y,S)be a fully stratified fuzzy rough topological space and $(Z,*'',R)$ fuzzy rough topological group. If $(Z,*'')$ is abelian group, then $(FRC(Y,Z), \odot)$ group is abelian too.

5. FUZZY ROUGH SHEAF

Let (X,T) be a fuzzy rough topological space as base set. Then it can be constituted a pointed fuzzy rough topological space (X, P_a^{λ}) with the same homotopy type for any fuzzy rough point $P_a^{\lambda} \in FRS(X)$. Furthermore $(Z,*'',R)$ fuzzy rough topological group with base fuzzy rough point $P_e^{\lambda} \in FRS(Z)$. Then it is denoted by $(Z,*'',P_e^{\lambda})$. If $(Z,*'',P_e^{\lambda})$ is any fuzzy rough topological group, then the fuzzy rough set of fuzzy rough homotopy class of homotopy maps preserving the base fuzzy rough points from (X,P_a^{λ}) to $(Z,*'',P_e^{\lambda})$, with respect to the fuzzy rough continuous base point, will be denoted by $H_a = (H_{a_L}, H_{a_U}) = [(X,P_a^{\lambda}),(Z,*'',P_e^{\lambda})] = \left\{[f]_{P_a^{\lambda}} \middle| f: (X,P_a^{\lambda}) \to (Z,*'',P_e^{\lambda}),\ f(P_a^{\lambda}) = P_e^{\lambda}, f \text{ is fuzzy rough continuous}\right\}$. In addition to membership function of $H_a = (H_{a_L}, H_{a_U})$ is $\mu_{H_{a_L}}: H_{a_L} \to I$ and $\mu_{H_{a_U}}: H_{a_U} \to I$, $\mu_{H_{a_L}}([f]_{P_{a_L(x)}^{\lambda}}) = \begin{cases} \lambda \text{ for } a_L = x \\ 0 \text{ for } a_L \neq x \end{cases}$ for every $x \in X_L$ and $\mu_{H_{a_U}}([f]_{P_{a_U(x)}^{\lambda}}) = \begin{cases} \lambda \text{ for } a_U = x \\ 0 \text{ for } a_U \neq x \end{cases}$ for every $x \in X_U$.

Property 5.1. Let $f, g \in FRC((X, P_a^\lambda), (Z, *'', P_e^\lambda))$ then the fuzzy rough function $(f \odot g)(x) = f(x) *'' g(x)$ for all $x \in X_L$ and $(f \odot g)(x) = f(x) *'' g(x)$ for all $x \in X_U$ preserves the base point.

Corollary 5.1. We can write $[f]_{P_a^\lambda} \cdot [g]_{P_a^\lambda} = [f \odot g]_{P_a^\lambda} \in [(X, P_a^\lambda), (Z, *'', P_e^\lambda)]$ for every $[f]_{P_a^\lambda}, [g]_{P_a^\lambda} \in [(X, P_a^\lambda), (Z, *'', P_e^\lambda)]$

Property 5.2. If (X, P_a^λ) pointed fuzzy topological space, $(Z, *'', P_e^\lambda)$ pointed fuzzy rough topological group, then $([(X, P_a^\lambda), (Z, *'', P_e^\lambda)], \cdot)$ is a group. In addition to that such a groups are also different from each other.

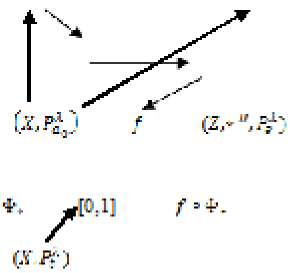
Property 5.3. Let (X, P_a^λ) pointed fuzzy topological space, $(Z, *'', P_e^\lambda)$ pointed fuzzy rough topological group. If $(Z, *'')$ group is Abelian, then $(H_a, \cdot)$ group is abelian.

Definition 5.1. Let X be a rough group and $G = (G_L, G_U)$ be a fuzzy rough set in X with the membership function $\mu_G = (\mu_{G_L}, \mu_{G_U})$ Then G is a *fuzzy rough group* in X iff the following conditions are satisfied.

$\mu_{G_L}(xy) \geq \min\{\mu_{G_L}(x), \mu_{G_L}(y)\}$ for every $x, y \in X_L$ and $\mu_{G_U}(xy) \geq \min\{\mu_{G_U}(x), \mu_{G_U}(y)\}$ for every $x, y \in X_U$ and $\mu_{G_L}(x^{-1}) \geq \mu_{G_L}(x)$ for every $x \in X_L$ and $\mu_{G_U}(x^{-1}) \geq \mu_{G_U}(x)$ for every $x \in X_U$.

Property 5.4. Let (X, P_a^λ) pointed fuzzy topological space, $(Z, *'', P_e^\lambda)$ pointed fuzzy rough topological group, then $H_a = [(X, P_a^\lambda), (Z, *'', P_e^\lambda)]$ is fuzzy rough group on X.

Let us denote by H the disjoint union of the fuzzy rough groups $[(X, P_a^\lambda), (Z, *'', P_e^\lambda)] = H_a$ obtained for each $P_a^\lambda \in FRS(X)$, (X, P_a^λ) pointed fuzzy rough topological space, i.e. $(H_L, H_U) = \cup_{P_a^\lambda \in FRS(X)} [(X, P_a^\lambda), (Z, *'', P_e^\lambda)] = \cup H_a$. Thus H is a fuzzy rough set over $FRS(X)$. Let us now define a function $\psi: H \to FRS(X)$ as $[f]_{P_a^\lambda} \in H \Rightarrow \exists a = (a_L, a_U) \in X$ such that $[f]_{P_a^\lambda} \in H_a \subseteq H \Rightarrow \psi([f]_{P_a^\lambda}) = P_a^\lambda \in FRS(X)$. For $a_0 = (a_{0_L}, a_{0_U}) \in X$ arbitrary fixed point, $W = W(P_{a_0}^\lambda)$ is fuzzy rough open Q neighbourhood of $P_{a_0}^\lambda$ in $FRS(X)$. Define a mapping $s: W \to H$ as follows: If $P_{a_0}^\lambda \in FRS(X)$ then there exists H_{a_0} rough group in H. Let $[f]_{P_{a_0}^\lambda}$ be the homotopy class in the rough group H_{a_0}. If P_t^λ is any fuzzy rough point in W, then (X, P_t^λ) and $(X, P_{a_0}^\lambda)$ are having the same homotopy type. Therefore, there is a homotopy equivalence function, $(X, P_t^\lambda) \xrightarrow{\Phi_*} (X, P_{a_0}^\lambda)$. Hence



From diagram functions f and Φ_* is fuzzy rough continuous and base point preserving, $f \circ \Phi_*$ composition is fuzzy rough continuous, too. In addition this composition is fuzzy rough preserves the base point, that is $(f \circ \Phi_*)(P_t^\lambda) = f\left(\Phi_*(P_t^\lambda)\right) = f(P_{a_0}^\lambda) = P_{f(a_0)}^\lambda = P_e^\lambda$. Hence $[h]_{P_t^\lambda} \in H_t$ is a homotopy class of function $f \circ \Phi_* = h$. In that case we can define s function as follows: $s: W \to H, s \to s(P_t^\lambda) = [h]_{P_t^\lambda}$ for every $P_t^\lambda \in W$.

In this way s is well defined. Therefore for all $P_t^\lambda \in W, (\psi \circ s)(P_t^\lambda) = \psi\left(s(P_t^\lambda)\right) = \psi\left([h]_{P_t^\lambda}\right) = P_t^\lambda$, then $\psi \circ s = I_W$. Hence we can write $s(W) = \cup_{P_t^\lambda \in W} [h]_{P_t^\lambda}$.

Remark 5.1. Let us denote the totality of the mapping s defined on W by $\Gamma(W, H)$.

Define $s(W)$ as an fuzzy rough open set, then it can be easily shown that the $\beta = \{s(W): W = W(P_a^\lambda) \subseteq FRS(X), s \in \Gamma(W, H)\}$ family is fuzzy rough topology base on H. Thus H is a fuzzy rough topological space. To show that $\psi: H \to FRS(X)$ is locally fuzzy rough topological mapping. If $[h]_{P_a^\lambda} \in H$ then $\psi([h]_{P_a^\lambda}) = P_a^\lambda$. Therefore there is a function $s: W \to H$ such that $s(P_t^\lambda) = [h]_{P_t^\lambda}, P_t^\lambda \in W = W(P_{a_0}^\lambda)$. Let us assume that $s(W) = R([h]_{P_a^\lambda})$ and $\psi|_R = \psi^*$.

- The function $\psi^* = \psi|_R: R \to W$ is injective. Because for any $[g]_{P_a^\lambda}, [h]_{P_a^\lambda} \in R = s(W)$ there are the fuzzy rough point P_a^λ, P_t^λ respectively in W such that $s(P_a^\lambda) = [g]_{P_a^\lambda} = [f \circ \Phi]_{P_a^\lambda}$, $s(P_t^\lambda) = [h]_{P_t^\lambda} = \left[f \circ \Phi'\right]_{P_t^\lambda}$. If $\psi^*\left([g]_{P_a^\lambda}\right) = \psi^*\left([h]_{P_t^\lambda}\right)$ then $\psi^*\left(s(P_a^\lambda)\right) = \psi^*\left(s(P_t^\lambda)\right) \Rightarrow \psi^*\left([f \circ \Phi]_{P_a^\lambda}\right) = \psi^*\left(\left[f \circ \Phi'\right]_{P_t^\lambda}\right) \Rightarrow P_a^\lambda = P_t^\lambda$ and $a = t$ where $a = (a_L, a_U), t = (t_L, t_U)$. Since (X, P_a^λ) with (X, P_t^λ) is same homotopy type, we can write the following equality, $\Phi \simeq \Phi' \Rightarrow f \circ \Phi \simeq f \circ \Phi' \Rightarrow [f \circ \Phi]_{P_a^\lambda} \simeq [f \circ \Phi']_{P_t^\lambda} \Rightarrow [g]_{P_a^\lambda} \simeq [h]_{P_t^\lambda}$.

- The function $\psi^* = \psi|_R: R \to W$ is fuzzy rough continuous. In fact, if $[h]_{P_a^\lambda} \in R = s(W)$ then $\psi^*\left([h]_{P_a^\lambda}\right) = P_a^\lambda \in W$ and $V = V(P_a^\lambda) \subset W$ is a Q neighbourhood of P_a^λ. Then $s(V) \subset R = s(W)$ is Q neighbourhood of $[h]_{P_a^\lambda}$ and $\psi^*(s(V)) = V \subset W$, ψ^* is fuzzy rough continuous.

- $(\psi^*)^{-1} = (\psi|_R)^{-1} = s: W \to R = s(W)$ is fuzzy rough continuous. In fact, if P_a^λ is any fuzzy rough point of W, $s(P_a^\lambda) = [h]_{P_a^\lambda} \in R'$ and $R' = R(P_a^\lambda) \subset R$ is a fuzzy rough Q neighbourhood of $[h]_{P_a^\lambda}$, then $(\psi|_R)(R') \subset W$ is fuzzy rough Q neighbourhood of P_a^λ in W and $s(\psi|_R)(R') = R' \subset R$. Therefore $(\psi^*)^{-1}$ is fuzzy rough continuous. From this, we can conclude that the following Definition 5.2.

Definition 5.2. Let $(Z, *'', R)$ be fuzzy rough topological group and let (X, P_a^λ) be pointed fuzzy rough topological space which have same homotopy type. If for every $P_a^\lambda \in FRS(X)$, $H = (H_L, H_U) = \cup_{P_a^\lambda \in FRS(X)}$

$[(X, P_a^\lambda), (Z, *'', P_e^\lambda)] = \cup H_a$ and $\psi: H \to FRS(X)$ is a function such that $\psi\left([h]_{P_a^\lambda}\right) \in H$ then there exists a fuzzy rough topology on H such that ψ is a locally fuzzy rough topological mapping with respect to this fuzzy rough topology. Thus the pair (H, ψ) is called *fuzzy rough sheaf* on $FRS(X)$.

Definition 5.3. The group $[(X, P_a^\lambda), (Z, *'', P_e^\lambda)] = \psi^{-1}(P_a^\lambda)$ is called the *stalk of the fuzzy rough sheaf* on $FRS(X)$ and is denoted by $H_{P_a^\lambda}$ for every $a \in X$. If $P_a^\lambda \in FRS(X)$ is an arbitrary fixed fuzzy rough point, then there is $W = W(P_a^\lambda)$ an fuzzy rough open Q neighbourhood of P_a^λ and function $s: W \to H$ such that s is fuzzy rough continuous and $\psi \circ s = I_W$. Hence the function s is called section of H over W, is denoted by $\Gamma(W, H)$.

Property 5.5. Let the set of all sections of H over W is $\Gamma(W, H)$ we can define

$(s_1 \circledast s_2)(P_\alpha^\lambda) = s_1(P_\alpha^\lambda) \cdot s_2(P_\alpha^\lambda) = [h]_{P_\alpha^\lambda} \cdot [g]_{P_\alpha^\lambda} = [h \circledast g]_{P_\alpha^\lambda}$ for any $s_1, s_2 \in \Gamma(W, H)$. Then $(\Gamma(W, H), \circledast)$ is a group. Hence (H, ψ) is an algebraic sheaf.

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